Challenger Entry and Electoral Accountability

Online Appendix

Jacob Morrier

Division of the Humanities and Social Sciences

California Institute of Technology

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A Statement of Proposition A1

Proposition A1. *In equilibrium, low-ability incumbents always implement policy a in the first period when the following condition is met:*

$$\frac{2\pi - 1}{\delta\pi} \geq (1 - q_i)(1 - q_c).$$

In this case, when the Voter does not observe either candidate's type before the election, they reelect the Incumbent after she implements policy a in the first period if $\kappa \geq \frac{\gamma}{\gamma+(1-\gamma)\pi}$, replace her with the Challenger otherwise, and reelect the Incumbent with certainty after she implements policy b.

If the equation does not hold, the equilibria of the model are as follows:

(i) If $\kappa < \gamma$:

- *Low-ability incumbents implement policy a with probability $\sigma = 1 - \frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}(1-\pi)$;*
- *When the Voter does not observe either candidate's type before the election, they replace the Incumbent with the Challenger with certainty after she implements policy a in the first period and reelect the Incumbent with probability $v^b = \frac{\frac{2\pi-1}{\delta\pi}}{(1-q_i)(1-q_c)}$ after she implements policy b;*

(ii) If $\kappa \in \left(\gamma, \frac{\gamma}{\gamma+(1-\gamma)\pi}\right)$:

- *Low-ability incumbents implement policy a with probability $\sigma = \frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}\pi$;*
- *When the Voter does not observe either candidate's type before the election, they reelect the Incumbent with probability $v^a = 1 - \frac{\frac{2\pi-1}{\delta\pi}}{(1-q_i)(1-q_c)}$ after she implements policy a in the first period and with certainty after she implements policy b;*

(iii) If $\kappa > \frac{\gamma}{\gamma+(1-\gamma)\pi}$:

- *Low-ability incumbents implement policy a with certainty;*
- *When the Voter does not observe either candidate's type before the election, they reelect the Incumbent with certainty independently of her policy decision in the first period.*

B Proof of Proposition 1

Proof. Conditional on the state of the world, it is sequentially rational for high-ability incumbents to implement the policy that matches the state of the world in the first period if and only if the following holds:

$$1 + \delta \times \bar{v}_i\left(h; \gamma, \nu^{\omega}\right) \times 1 \geq 0 + \delta \times \bar{v}_i\left(h; \gamma, \nu^{-\omega}\right) \times 1.$$

Here, in a minor notational simplification, I use $-\omega$ to represent the policy opposite to the state ω.

This equation can be rearranged as follows:

$$\delta \times \left(\bar{v}_i\left(h; \gamma, \nu^{-\omega}\right) - \bar{v}_i\left(h; \gamma, \nu^{\omega}\right)\right) \leq 1. \tag{B1}$$

This equation stipulates that when pondering which policy to implement in the first period, high-ability incumbents weigh the cost of implementing the "wrong" policy against the resulting improvement in their reelection prospects.

Since it is the difference between two probabilities, the improvement in high-ability incumbents' reelection prospects resulting from implementing the policy that does not match the state of the world is at most one:

$$\bar{v}_i\left(h; \gamma, \nu^{-\omega}\right) - \bar{v}_i\left(h; \gamma, \nu^{\omega}\right) \leq 1.$$

Combined with the assumption that the discount factor δ has a value strictly less than one, it follows that Equation (B1) holds with strict inequality:

$$\delta \times \left(\bar{v}_i\left(h; \gamma, \nu^{-\omega}\right) - \bar{v}_i\left(h; \gamma, \nu^{\omega}\right)\right) \leq \delta \times 1 < 1.$$

Consequently, in equilibrium, high-ability incumbents necessarily implement the policy that matches the state of the world in the first period. □

C Proof of Proposition 2

Proof. It is sequentially rational for low-ability incumbents to implement policy a in the first period if and only if the following holds:

$$\pi \times 1 + (1-\pi) \times 0 + \delta \times \bar{v}_i \left(\ell; \gamma, v^a\right) \times \pi \geq \pi \times 0 + (1-\pi) \times 1 + \delta \times \bar{v}_i \left(\ell; \gamma, v^b\right) \times \pi.$$

This equation can be rearranged as follows:

$$\bar{v}_i \left(\ell; \gamma, v^b\right) - \bar{v}_i \left(\ell; \gamma, v^a\right) \leq \frac{2\pi - 1}{\delta \pi}. \tag{C1}$$

This equation stipulates that, when pondering which policy to implement in the first period, low-ability incumbents weigh the cost of implementing policy b, which is less likely to match the state of the world than policy a, against the value of the resulting improvement in their reelection prospects.

Leveraging the results in Section 3, it is easily shown that the left-hand side of Equation (C1) equals:

$$\bar{v}_i \left(\ell; \gamma, v^b\right) - \bar{v}_i \left(\ell; \gamma, v^a\right) = (1-q_i)(1-q_c)\left(v^b - v^a\right).$$

Sequential rationality of the Voter's electoral behavior imposes that, absent any exogenous information about candidates' private types before the election, they must elect the candidate who is most likely to have a high ability:

$$v^y = 1\,(0) \Rightarrow \kappa^y \geq (\leq)\, \gamma.$$

Since it is the difference between two probabilities, the improvement in low-ability incumbents' reelection probability absent any exogenous information about candidates' private types before the election is at most one:

$$v^b - v^a \leq 1.$$

It follows that if $(1-q_i)(1-q_c) \leq \frac{2\pi-1}{\delta\pi}$, Equation (C1) necessarily holds, reflecting the fact that the cost of carrying out policy b in the first period systematically outweigh the potential improvements in low-ability incumbents' reelection prospects. In this case, low-ability incumbents implement policy a in the first period with certainty. In contrast, if $(1-q_i)(1-q_c) > \frac{2\pi-1}{\delta\pi}$, the potential improvements in low-ability incumbents' reelection prospects may be sufficiently valuable for them to distort their policy decisions in equilibrium.

In equilibrium, the difference in low-ability incumbents' reelection probabilities conditional on their first-period policy decisions must be less than or equal to the cost of pursuing policy b in the first period relative to the benefits of holding office in the second period. Put formally, Equation (C1) must be satisfied. To prove this, let us assume it was not. In that case, low-ability incumbents would find it beneficial to implement policy b invariably in the first period. Accordingly, if she enacted policy a, the Voter would deduce that the Incumbent has a high ability. This would negate the electoral benefits associated with policy b, thus eliminating the Incumbent's motives for distorting her policy choices in the first place.

Henceforth, we distinguish two cases: whether Equation (C1) holds with strict inequality or with equality in equilibrium.

In the first case, low-ability incumbents necessarily implement policy a in the first period. Accordingly, the Voter infers that if she implements policy b in the first period, the Incumbent has a high ability, ensuring her reelection, while if she implements policy a, she has a probability $\kappa^a = \frac{\kappa\pi}{\kappa\pi+(1-\kappa)}$ of having high ability.

Since I have assumed that low-ability incumbents found the potential improvements in their reelection prospects from implementing policy b sufficiently valuable, it will be sequentially rational for them to systematically implement policy a in the first period only if they are guaranteed to be reelected after doing so. For this to occur in equilibrium, the Incumbent must be sufficiently likely to have a high ability after implementing policy a in the first period so that, absent any exogenous information disclosure, the Voter finds it sequentially rational to reelect the Incumbent rather than replace her with the Challenger:

$$v^a = 1 \Rightarrow \kappa^a \geq \gamma.$$

The latter condition can be reformulated as follows:

$$\frac{\kappa\pi}{\kappa\pi + (1-\kappa)} \geq \gamma \Leftrightarrow \kappa \geq \frac{\gamma}{\gamma + (1-\gamma)\,\pi}.$$

In the second case, low-ability incumbents are indifferent between both available policies. Accordingly, they randomize between pursuing both in the first period. The extent to which they do is set to make the Voter indifferent between reelecting the Incumbent or replacing her with the Challenger after she has implemented one of the two available policies:

$$\kappa^y = \gamma.$$

In turn, after she has implemented one of the two available policies, the Voter must randomize between reelecting the Incumbent and replacing her with the Challenger to make low-ability incumbents indifferent between both policies:

$$\left(1-q_i\right)\left(1-q_c\right)\left(v^b-v^a\right)=\frac{2\pi-1}{\delta\pi}.$$

Note that, for this equation to hold, the Incumbent must be strictly more likely to be reelected after implementing policy b relative to policy a, or, formally, we must have $v^b>v^a$. In turn, sequential rationality of the Voter's electoral choices requires that we have $\kappa^b>\kappa^a$.

Given that the Voter can only randomize between reelecting the Incumbent and replacing her with the Challenger after she has implemented one of the policies in equilibrium, there are two subcases to consider: the one in which the Voter is indifferent between reelecting the Incumbent and replacing her with the Challenger after she has implemented policy a, and the other in which they are indifferent after the Incumbent has implemented policy b.

In the first subcase, low-ability incumbents must implement policy a with probability σ to make the posterior probability that the Incumbent has a high ability conditional on implementing policy a equal to the Challenger's probability of having a high ability:

$$\kappa^a=\gamma\Leftrightarrow\frac{\kappa\pi}{\kappa\pi+\left(1-\kappa\right)\sigma}=\gamma\Leftrightarrow\sigma=\frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}\pi.$$

Demonstrably, this value of σ is strictly positive. This value also needs to be lower than one, which translates into the following condition:

$$\frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}\pi\leq1\Leftrightarrow\kappa\leq\frac{\gamma}{\gamma+\left(1-\gamma\right)\pi}.$$

To ensure sequential rationality of the Voter's electoral choices, the Incumbent must have a higher probability of having a high ability conditional on having implemented policy b in the first period than the Challenger:

$$\kappa^b\geq\gamma\Leftrightarrow\frac{\kappa\left(1-\pi\right)}{\kappa\left(1-\pi\right)+\left(1-\kappa\right)\left(1-\sigma\right)}\geq\gamma\Leftrightarrow\kappa\geq\gamma.$$

In the second subcase, low-ability incumbents must implement policy b with probability σ to make the posterior probability that the Incumbent has a high ability conditional on implementing policy b equal to the Challenger's probability of having a high ability:

$$\kappa^b=\gamma\Leftrightarrow\frac{\kappa\left(1-\pi\right)}{\kappa\left(1-\pi\right)+\left(1-\kappa\right)\left(1-\sigma\right)}=\gamma\Leftrightarrow\sigma=1-\frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}\left(1-\pi\right).$$

Demonstrably, this value of σ is strictly below one. This value also needs to be positive, which translates into the following condition:

$$1 - \frac{\kappa}{1-\kappa}\frac{1-\gamma}{\gamma}(1-\pi) \geq 0 \Leftrightarrow \kappa \leq \frac{\gamma}{\gamma + (1-\gamma)(1-\pi)}.$$

To ensure sequential rationality of the Voter's electoral choices, the Incumbent must have a lower probability of having a high ability after implementing policy a in the first period than the Challenger:

$$\kappa^a \leq \gamma \Leftrightarrow \frac{\kappa\pi}{\kappa\pi + (1-\kappa)\sigma} \leq \gamma \Leftrightarrow \kappa \leq \gamma.$$

Note that this condition is the only binding one, as the previous one necessarily holds if it does. $\square$

D Proof of Lemma 1

Proof. Equation (2) characterizes the circumstances under which it is sequentially rational for the Challenger to run for office:

$$\kappa^y \leq \frac{\gamma\left(1-\left(1-q_i\right)\left(1-q_c\right)\right)+\left(1-\gamma\right)\pi q_i\left(1-q_c\nu_\ell\right)+\left(\gamma+\left(1-\gamma\right)\pi\right)\left(1-q_i\right)\left(1-q_c\right)\left(1-\nu^y\right)-c}{q_i\left(\gamma\left(1-q_c\left(1-\nu_h\right)\right)+\left(1-\gamma\right)\pi\left(1-q_c\nu_\ell\right)\right)}.$$

However, the equation does not directly characterize the Challenger's equilibrium entry strategy. The reason is that both sides depend on κ^y. Indeed, the left-hand side contains it explicitly, whereas the right-hand side contains ν^y, which indirectly depends on κ^y through the sequential rationality of the Voter's electoral behavior. In particular, sequential rationality stipulates that the Voter must elect the candidate who is most likely to have a high ability to hold office in the second period:

$$\nu^y = 1\;(0) \Rightarrow \kappa^y \geq (\leq)\,\gamma.$$

To characterize the Challenger's equilibrium entry strategy, it is necessary to examine three scenarios contingent on the value of ν^y.

Firstly, if $\nu^y = 1$, the right-hand side of Equation (2) equals $\frac{\gamma(1-(1-q_i)(1-q_c))+(1-\gamma)\pi q_i(1-q_c\nu_\ell)-c}{q_i(\gamma(1-q_c(1-\nu_h))+(1-\gamma)\pi(1-q_c\nu_\ell))}$, which I henceforth denote by $\underline{\kappa}$. In this case, sequential rationality of the Voter's electoral choices further requires that we have $\kappa^y \geq \gamma$. Both conditions cannot concurrently hold unless $\gamma < \underline{\kappa}$. In this case, the Challenger runs for office if and only if $\kappa^y \in \left(\gamma, \underline{\kappa}\right)$. If $\kappa^y = \underline{\kappa}$, the Challenger may randomize between entry decisions as he is indifferent between running and not running.

Secondly, if $\nu^y = 0$, the right-hand side of Equation (2) equals $\frac{\gamma+(1-\gamma)\pi(q_i(1-q_c\nu_\ell)+(1-q_i)(1-q_c))-c}{q_i(\gamma(1-q_c(1-\nu_h))+(1-\gamma)\pi(1-q_c\nu_\ell))}$, which I henceforth denote by $\bar{\kappa}$. In this case, sequential rationality of the Voter's electoral choices further requires that we have $\kappa^y \leq \gamma$. Accordingly, the Challenger runs for office if and only if $\kappa^y \leq \min\{\gamma, \bar{\kappa}\}$. If $\bar{\kappa} < \gamma$, this means that the Challenger runs for office if and only if $\kappa^y \leq \bar{\kappa}$. Also, if $\kappa^y = \bar{\kappa}$, the Challenger may randomize between entry decisions as he is indifferent between running or not. On the other hand, if $\bar{\kappa} > \gamma$, this means that the Challenger runs for office if and only if $\kappa^y \leq \gamma$.

Thirdly, if $\nu^y \in (0,1)$, the right-hand side of Equation (2) equals something between $\underline{\kappa}$ and $\bar{\kappa}$. Sequential rationality of the Voter's electoral choices requires that we have $\kappa^y = \gamma$. Generically, this occurs only if $\sigma \in (0,1)$. This requires that low-ability incumbents be indifferent between implementing both policies in the first period and that ν^y be defined as such. In this case, the Challenger runs for office if and only if the probability that the Challenger has high ability is lower than or equal to the value of the right-hand side of Equation (2) induced by that value of ν^y. The value of ν^y may also be set to make the Challenger indifferent between running and not running:

$$\tilde{\nu} = \frac{q_i \left(\gamma \left(1 - q_c \left(1 - \nu_h\right)\right) + \left(1 - \gamma\right) \pi \left(1 - q_c \nu_\ell\right)\right)}{\left(\gamma + \left(1 - \gamma\right) \pi\right) \left(1 - q_i\right) \left(1 - q_c\right)} \left(\bar{\kappa} - \gamma\right).$$

In this case, the Challenger may randomize between running and not running, and he must do so to make the Voter indifferent between reelecting the Incumbent and replacing her with the Challenger absent any exogenous information disclosure. □

For clarity of exposition, Figure D1 illustrates the Challenger's equilibrium entry strategy, with the areas over which the Challenger runs for office either crosshatched or shaded. The x-axis represents the probability that the Challenger has a high ability. The y-axis represents the posterior probability that the Incumbent has a high ability conditional on her first-period policy decision. The crosshatched area highlights the case wherein: (i) the Challenger runs for office, and (ii) the Incumbent is necessarily reelected absent exogenous information disclosure. The shaded area highlights the case wherein: (i) the Challenger runs for office, and (ii) he necessarily replaces the Incumbent absent exogenous information disclosure.

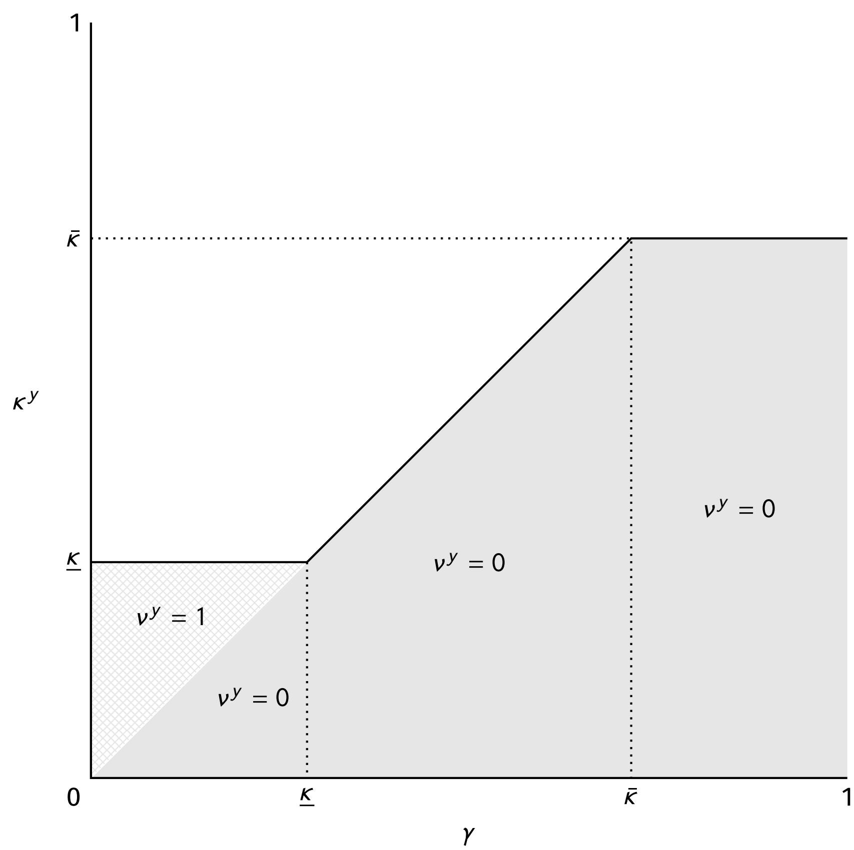

Figure D1: The Challenger's Equilibrium Entry Strategy

E Proof of Lemma 2

Proof. Firstly, I consider the case in which $\gamma < \underline{\kappa}$.

If $\kappa^y < \gamma$, the Voter replaces the Incumbent with the Challenger when there is no exogenous information disclosure. Since $\gamma < \underline{\kappa} < \bar{\kappa}$, it is sequentially rational for the Challenger to run for office. It follows that low-ability incumbents' reelection probability is $(1-\gamma)\, q_c\, (1 - q_i\, (1-\nu_\ell))$. Note that if $\kappa^y = \gamma$, all values in the interval $[(1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell))\, , (1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell)) + (1-q_i)\, (1-q_c)]$ can be sustained in equilibrium since the Voter is indifferent between reelecting the Incumbent and replacing her with the Challenger.

On the other end of the spectrum, if $\kappa^y > \bar{\kappa}$, it is sequentially rational for the Challenger not to run for office. Since $\gamma < \underline{\kappa} < \bar{\kappa}$, it is sequentially rational for the Voter to reelect the Incumbent absent any exogenous information disclosure. It follows that the Incumbent is reelected with certainty. Note that if $\kappa^y = \bar{\kappa}$, all values in the interval $[(1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell)) + (1-q_i)\, (1-q_c)\, , 1]$ can be sustained in equilibrium since the Challenger is indifferent between running or not.

If $\kappa^y \in (\gamma, \underline{\kappa})$, it is sequentially rational for the Challenger to run for office and for the Voter to reelect the Incumbent absent any information about candidates' types before the election. Also, the Voter reelects the Incumbent without exogenous information disclosure before the election. It follows that low-ability incumbents' reelection probability is $(1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell)) + (1-q_i)\, (1-q_c)$.

Secondly, I consider the case in which $\gamma \in (\underline{\kappa}, \bar{\kappa})$.

If $\kappa^y < \gamma$, it is sequentially rational for the Challenger to run for office and the Voter to replace the Incumbent with the Challenger absent any information about the candidates' types before the election, in which case low-ability incumbents' reelection probability equals $(1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell))$. On the other hand, if $\kappa^y > \gamma$, it is sequentially rational for the Challenger to concede to the Incumbent and the Voter to reelect the Incumbent absent any information about the candidates' types before the election, in which case low-ability incumbents' reelection is assured.

I now consider the case when $\kappa^y = \gamma$. In this case, I show that there are equilibrium values of ν^y and ρ^y such that low-ability incumbents' reelection probability can take any value in the interval $[(1-\gamma)\, q_c\, (1-q_i\, (1-\nu_\ell))\, , 1]$ in equilibrium. Note that since $\gamma \in (\underline{\kappa}, \bar{\kappa})$, there is a value of $\nu^y \in (0,1)$ such that the right-hand side of Equation (2) equals to γ. I denote this value as $\tilde{\nu}$:

$$\tilde{\nu} = \frac{q_i\, (\gamma\, (1-q_c\, (1-\nu_h)) + (1-\gamma)\, \pi\, (1-q_c\nu_\ell))}{(\gamma + (1-\gamma)\, \pi)\, (1-q_i)\, (1-q_c)}\, (\bar{\kappa} - \gamma)\, .$$

If $\nu^y < \tilde{\nu}$, the Challenger necessarily runs for office in equilibrium. In contrast, if $\nu^y > \tilde{\nu}$, the Challenger surrenders to the Incumbent.

Given that $\kappa^y = \gamma$, the Voter is indifferent between reelecting the Incumbent and replacing her with the Challenger. Thus, all values of $\nu^y \in (0,1)$ can be sustained in equilibrium. Furthermore, if $\nu^y \leq \bar{\nu}$, it is sequentially rational for the Challenger to run for office. It follows that all values in the interval $[(1-\gamma)\, q_c\, (1-q_i\,(1-\nu_\ell))\,, (1-\gamma)\, q_c\, (1-q_i\,(1-\nu_\ell)) + (1-q_i)\,(1-q_c)\,\bar{\nu}]$ can be sustained as low-ability incumbents' reelection probability in equilibrium.

If $\kappa^y = \gamma$ and $\nu^y = \bar{\nu}$, then the Challenger is indifferent between running for office and conceding to the Incumbent. In that case, all values of $\rho^y \in (0,1)$ are sustainable in equilibrium. This implies that all values in the interval $[(1-\gamma)\, q_c\, (1-q_i\,(1-\nu_\ell)) + (1-q_i)\,(1-q_c)\,\bar{\nu}, 1]$ can be sustained as equilibrium values of low-ability incumbents' reelection probability.

Thirdly, I consider the case in which $\gamma > \bar{\kappa}$.

If $\kappa^y < \bar{\kappa}$, it is sequentially rational for the Challenger to run for office. Also, since $\gamma > \bar{\kappa}$, the Voter replaces the Incumbent with the Challenger absent any information about candidates' private types before the election. It follows that low-ability incumbents' reelection probability is $(1-\gamma)\, q_c\, (1-q_i\,(1-\nu_\ell))$.

If $\kappa^y = \bar{\kappa}$, the Challenger is indifferent between running or not. Therefore, all values in the interval $[(1-\gamma)\, q_c\, (1-q_i\,(1-\nu_\ell))\,, 1]$ can be sustained as equilibrium values of low-ability incumbents' reelection probability.

Finally, if $\kappa^y > \bar{\kappa}$, it is sequentially rational for the Challenger not to run for office. This is true if $\kappa^y < \gamma$ and the Voter replaces the Incumbent without the Challenger without exogenous information disclosure before the election and even more if $\kappa^y > \gamma$ and the Voter reelects the Incumbent without exogenous information disclosure before the election. It follows that low-ability incumbents are reelected with certainty. $\square$

F Proof of Proposition 4

Proof. The Voter's welfare equals the sum of the expected policy payoffs in the first and second periods. The expected payoffs induced by the Incumbent's equilibrium policy decision in the first period equal:

$$\kappa \times 1 + (1-\kappa) \times (\sigma \times \pi + (1-\sigma) \times (1-\pi)) .$$

These equal the sum of the probability that the Incumbent has a high ability multiplied by one and the probability that the Incumbent has a low ability times the expected payoffs from low-ability incumbents' equilibrium policy decisions in the first period. The latter is equal to the probability that low-ability incumbents implement policy a in equilibrium times π, that is, the probability that policy a is the right policy, plus the probability that low-ability incumbents implement policy b in equilibrium times $1-\pi$, which is the probability that policy b is the right policy.

All else equal, expected first-period policy payoffs are inversely proportional to the probability that the Incumbent distorts her policy decisions. Thus, the effect of endogenous Challenger entry on first-period policy payoffs is straightforward to evaluate: if it leads to more policy distortions, endogenous Challenger entry decreases expected first-period policy payoffs, whereas if it leads to less policy distortions, endogenous Challenger entry increases expected first-period policy payoffs.

The effect of endogenous Challenger entry on the expected second-period policy payoffs is more difficult to ascertain. The expected policy payoffs in the second period are equal to the probability that the second-period officeholder has a high ability multiplied by one plus the likelihood that the second-period officeholder has a low ability times π. This implies that expected policy payoffs in the second period are proportional to the probability that the second-period officeholder has a high ability.

The probability that the second-period officeholder has a high ability equals:

$$\underbrace{(\kappa \times \pi + (1-\kappa) \times \sigma)}_{\text{Incumbent implements policy } a} \times (\rho^a \times \max\{\kappa^a, \gamma\} + (1-\rho^a) \times \kappa^a)$$
$$+ \underbrace{(\kappa \times (1-\pi) + (1-\kappa) \times (1-\sigma))}_{\text{Incumbent implements policy } b} \times \left(\rho^b \times \max\left\{\kappa^b, \gamma\right\} + \left(1-\rho^b\right) \times \kappa^b\right) .$$

Conditional on the policy implemented by the Incumbent in the first period, the probability that the second-period officeholder has a high ability equals the maximum of the posterior probability that the Incumbent has a high ability and the likelihood that the Challenger does if the latter runs for office. On the other hand, if the Challenger does not run, the probability that the second-period officeholder has a high ability equals the posterior probability that the Incumbent does. Therefore, the second-period officeholder is, all else equal, less likely to have a high ability when the Challenger is deterred from running for office.

Although this suggests that endogenous Challenger entry systematically lowers the probability that the second-period officeholder has a high ability, it is difficult to precisely assess the extent to which it does, given that the posterior probability that the Incumbent has a high ability also varies because of endogenous Challenger entry. Nonetheless, should the Voter reelect the Incumbent irrespective of her first-period policy decision, even when the Challenger always runs for office, endogenous Challenger entry does not impose any adverse effects. By construction, the probability that the second-period officeholder has a high ability is greater than or equal to the probability that the Incumbent has a high ability. Accordingly, if the former equals the latter when the Challenger always runs for office, then endogenous Challenger entry does not reduce the probability that the second-period officeholder has a high ability. Informally, the reason for this is that endogenous Challenger entry deprives the Voter of the option of replacing the Incumbent with the Challenger, an opportunity they were not prevailing themselves of in the first place.

Leveraging this observation, I identify sufficient conditions under which endogenous Challenger entry necessarily improves the Voter's welfare as it lowers policy distortions while keeping constant the probability that the second-period officeholder has a high ability. In Corollary 2, I show that endogenous Challenger entry lessens policy distortions compared to when the Challenger always runs if $\gamma > \bar{\kappa}$ and $\kappa \in \left(\frac{\gamma\bar{\kappa}}{\pi\gamma+(1-\pi)\bar{\kappa}}, \frac{\gamma}{\gamma+(1-\gamma)\pi}\right)$. Additionally, if $\kappa > \gamma$, the Voter reelects the Incumbent irrespective of her first-period policy decision, even when the Challenger always runs for office. It follows that if $\gamma > \bar{\kappa}$ and $\kappa \in \left(\gamma, \frac{\gamma}{\gamma+(1-\gamma)\pi}\right)$, endogenous Challenger entry improves the Voter's welfare relative to the scenario in which the Challenger always runs for office. $\square$